ELECTRICAL AND COMPUTER ENGINEERING
PhD QUALIFYING EXAMINATION

Session 1

August 20, 2007

Be sure to put your ID number on each sheet that has material to be graded. Do not put your name on any sheet.

There are 14 equally weighted problems. You are to SELECT ANY EIGHT of these to answer. You must make it very clear which eight that you choose. (If it is not clear, then the first eight problems that you attempt will be graded.) Indicate your selections in two ways:

1. Circle below which eight problems that you want graded.
2. If you write anything other than your ID number on the page of a question that you do not graded, the cross out that page with a large X from corner to corner.

Circle the eight questions that you want graded:

1

2

3

4

5

6

7

8

9

10

11

12

13

14

Do all work on the paper supplied to you. Do not write on the back of any page.

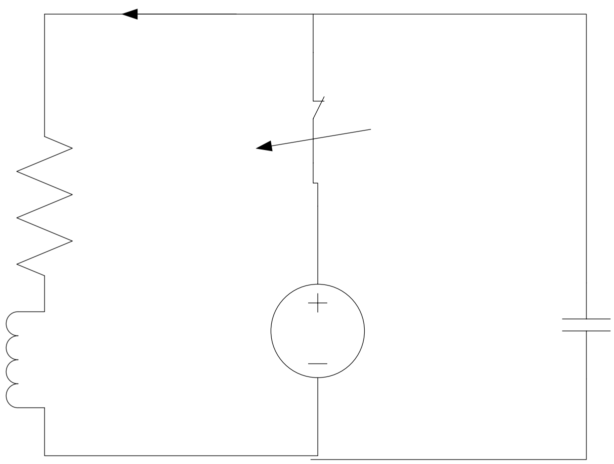
Problem 1-1 ID # ________________________

Design a logic circuit, which implements the following truth table of a 2-bit subtractor. The 2-bit subtractor subtracts a 2-bit number binary (y_2y_1) from a 2-bit binary number (x_2x_1) and outputs a 3-bit binary number ($s_3s_2s_1$).

x_2	x_1	y_2	y_1	s_3	S_2	s_1
0	0	0	0	0	0	0
0	0	0	1	1	1	1
0	0	1	0	1	1	0
0	0	1	1	1	0	1
0	1	0	0	0	0	1
0	1	0	1	0	0	0
0	1	1	0	1	1	1
0	1	1	1	1	1	0
1	0	0	0	0	1	0
1	0	0	1	0	0	1
1	0	1	0	0	0	0
1	0	1	1	1	1	1
1	1	0	0	0	1	1
1	1	0	1	0	1	0
1	1	1	0	0	0	1
1	1	1	1	0	0	0

Problem 2-1 ID # ____________________________

The switch has been closed for a long time and is opened at t = 0. Find the expression for i(t), t > 0.



Problem 3-1 ID # ________________________

3. A signal $x(t)$ has a continuous-time Fourier transform $X(f)$ that is non-zero only in the frequency range $58 < |f| < 64$. Assuming the signal's bandwidth and center frequency are known and the signal is sampled for all time by an ideal sampler, what is the absolute minimum exact sampling rate for which the signal could be recovered from the samples? (The answer is not 132. It is somewhere between 12 and 14.)

Problem 4-1 ID # ______________________

Expand $\frac{1}{(1+x)^2}$ as a power series, $|x| < 1$.

Problem 5-1 ID # ___________________________

Explain why a recursive implementation of the Fibonacci() function defined by

a. Fibonacci(0) = 1
b. Fibonacci(1) = 1
c. Fibonacci(n) = Fibonacci(n-1) + Fibonacci(n-2)

for all nonnegative n is not wise.

Problem 6-1 ID # ____________________________

n balls are thrown into n bins uniformly at random, that is, each ball falls into one of the n bins chosen independently and uniformly at random. Let P_n^k denote the probability that bin 1 has exactly k balls. For any fixed (and finite) k, $P^k = \lim_{n\to\infty} P_n^k$.

(a) Find P_n^0 for each $n \geq 1$, hence obtain P^0.

(b) Find P_n^k for $k \leq 10$ and for $n > k$. Obtain P^k for $k \leq 10$.

(c) What is the average fraction of the bins that remain empty as $n \to \infty$?

Problem 7-1 ID # ________________________

For the memory reference string (page addresses) shown below, indicate which references result in a page fault for the page replacement algorithms: FIFO (First-In-First-Out), OPTIMAL and LRU (full Least Recently Used). Assume that there are three physical page frames and there is only one process.

1. For each memory reference shown, place an **F** in the cell below the memory reference if that reference results in a page fault (for the algorithm). Assume there is nothing in physical memory when the reference string begins.

Reference String	0	1	2	0	1	2	3	4	2	5	0	6	0	7	7	8	0	6
FIFO																		
OPTIMAL																		
LRU																		

2. For the same memory reference sequence, complete this table.

Page referenced	Pages in memory when the reference is made		
	FIFO	OPTIMAL	LRU
0	nothing	nothing	nothing
1	0	0	0
2	01	01	01
0	012	012	012
1	012	012	120
2	012	012	201
3			
4			
2			
5			
0			
6			
0			
7			
7			
8			
0			
6			

Problem 8-1 ID # ______________________

An important problem in numerical analysis is to find a solution to the equation F(X) = 0 for some arbitrary F. If the function is continuous and has two points *low* and *high* such that F(*low*) and F(*high*) have opposite signs, then a root must exist between *low* and *high* and can be found by either a binary search or an interpolation search. Write a function that takes as parameters F, *low*, and *high* and solves for a zero. What must you do to ensure termination?

Problem 9-1 ID # ___________________________

One of the most studied problems in process synchronization and communication is the readers and writers problem. Let R = $\{r_1 \ldots r_m\}$ be a set of processes that read from a database. Let W = $\{w_1 \ldots w_k\}$ be a set of processes that write values to the same database. The system is subject to the following constraints:

1. Readers and writers can never access the database simultaneously;
2. Only a single writer at a time is allowed in the database;
3. Multiple readers may access the database simultaneously;

The following pseudo code is a version of a Reader-Writer solution. What may happen if we run this Reader-Writer solution (assume multiple instances of both Writers and Readers)? Explain your answer and improve this given solution.

```
//shared variables
Semaphore mutex(1), db(1);
int readcount = 0;                //number of processes reading or wanting to read

 writer ()
{
    while(TRUE){
        db.P()
        write_data_base();
        db.V();
        sleep_for_some_random_time();
      }
 }

 reader ()
{
   while(TRUE){
     mutex.P();
     readcount = readcount + 1;
     if (readcount==1)
           db.P();
     read_data_base();
     readcount = readcount - 1;
     if (readcount==0)
           db.V();
     mutex.V();
     sleep_for_some_random_time();
   }
}
```

Problem 10-1 ID # ________________________

A vector can be represented by a linear combination of the coordinates $c_1, c_2, \cdots, c_k$ and the basis vectors $\boldsymbol{u}_1, \boldsymbol{u}_2, \cdots, \boldsymbol{u}_k$:

$$\boldsymbol{x} = c_1\boldsymbol{u}_1 + c_2\boldsymbol{u}_2 + \cdots + c_k\boldsymbol{u}_k$$
$$= \begin{bmatrix} \boldsymbol{u}_1 & \boldsymbol{u}_2 \cdots \boldsymbol{u}_k \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_k \end{bmatrix}$$

Let

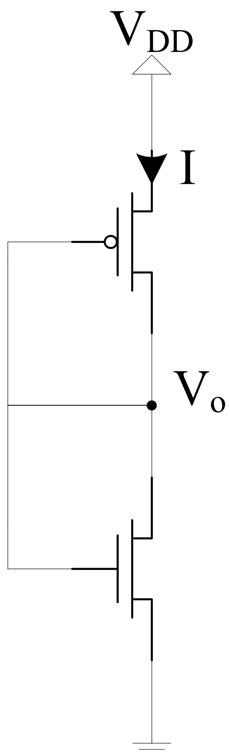
$$E = \begin{bmatrix} \boldsymbol{v}_1 & \boldsymbol{v}_2 & \boldsymbol{v}_3 \end{bmatrix} = \begin{bmatrix} (1,1,1)^T, (2,3,2)^T, (1,5,4)^T \end{bmatrix}$$
$$F = \begin{bmatrix} \boldsymbol{u}_1 & \boldsymbol{u}_2 & \boldsymbol{u}_3 \end{bmatrix} = \begin{bmatrix} (1,1,0)^T, (1,2,0)^T, (1,2,1)^T \end{bmatrix}$$

(a) Find the transition matrix from E to F.

(b) If $\boldsymbol{x} = 3\boldsymbol{v}_1 + 2\boldsymbol{v}_2 - \boldsymbol{v}_3$ and $\boldsymbol{y} = \boldsymbol{v}_1 - 3\boldsymbol{v}_2 + 2\boldsymbol{v}_3$, find the coordinates of $\boldsymbol{x}$ and $\boldsymbol{y}$ with respect to the ordered basis F using the transition matrix found in (a).

Problem 11-1 ID # ________________________

For the following MOSFET question, use these parameters:



	NMOS	PMOS
V_{TO}	0.75V	-0.75V
γ	$0.75\sqrt{V}$	$0.5\sqrt{V}$
$2\varphi_F$	0.6V	0.6V
K'	$25\mu A/V^2$	$10\mu A/V^2$

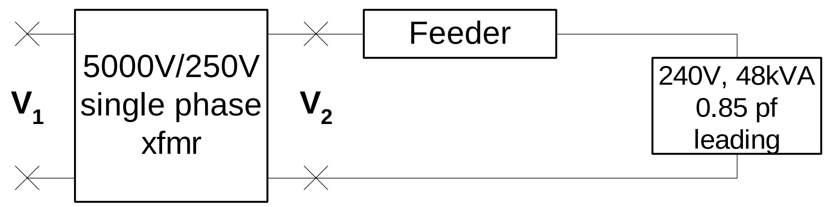
a. Label each of the terminals of the 2 MOSFETs on the diagram to the left.

b. Find the current *I* and the voltage V_o if W/L = 20/1 for both transistors and V_{DD} = 10V.

c. What is the current *I* if the W/L ratio for both transistors changes to 80/1?

Problem 12-1 ID # ________________________

For a single-phase transformer that has voltage ratings of 5000V/250V, the equivalent impedance when referred to the low voltage winding is $R_{eq2} + jX_{eq2} = 0.02 + j0.05\Omega$. A feeder between the transformer and a load has an equivalent impedance of $0.03 + j0.08\Omega$. The feeder supplies a 48 kVA load with a voltage of 240V at 0.85 power factor leading. Assume that the voltage across the load is the reference voltage for this problem. Find the following:



(a) Complex voltage $\vec{V}_2$ at the terminals of the transformer secondary.

(b) Complex voltage $\vec{V}_1$ at the terminals of the transformer primary.

Problem 13-1 ID # ________________________

Consider a plane wave obliquely incident from a half-space of a lossless dielectric with relative dielectric constant of $\varepsilon_r=3$ to air, and has a parallel polarization,

a) Find the reflection coefficient at the interface
b) Find the transmission coefficient
c) Find the condition of total reflection; i.e. $|\Gamma|=1$
d) Calculate the complex pointing vector for the surface fields, i.e. along the air-dielectric interface.

Problem 14-1 ID # ________________________

You are transmitting a binary signal such that a logic zero is represented by –2 volts and a logic 1 is represented by +2 volts. The signal is corrupted by additive noise. The noise has a probability density function (pdf) of $p_X(x) = 0.5e^{-|x|}$. Your detector is designed so that if a received signal has a value greater than 0 volts, it is interpreted as a logic 1 and if it is less than 0 volts it is interpreted as a logic 0. In other words the decision threshold is 0 volts.

a. Sketch a labeled graph of the noise pdf.

b. Sketch a labeled graph showing signal's pdf when a logic 1 is transmitted.

c. You are transmitting the same binary signal as given, except now the signal is corrupted by uniformly distributed noise that ranges from –4 volts to +4 volts. What is the probability of error when a logic 1 is transmitted? Suggestion: sketch the signal's pdf when a logic 1 is transmitted.

Problem _-_ ID # ___________________________