



DEPARTMENT OF ELECTRICAL ENGINEERING AND COMPUTER SCIENCE  
UNIVERSITY OF TENNESSEE  
SPRING 2011 Ph.D. QUALIFYING EXAMINATION  
Monday, January 10, 2011

Exam Packet Number: \_\_\_\_\_

You are allowed 4 hours to complete this exam.

- This exam is closed book and closed notes. No calculators or cell phones are allowed (other than the calculators provided to you by the proctors).
- All your work should be done on the papers that are supplied to you. Do not write on the back of any page. Do not write any answers on this packet!**
- Be sure to put your exam packet number on each sheet that has material to be graded. Do not put your name on any sheet!
- There are 16 equally weighted problems. You are to SELECT ANY EIGHT of these to answer. You must make it very clear which eight you choose (see below). If it is not made clear by you, then the first eight problems that you attempt to answer will be graded. Circle only the eight (8) questions that you want graded below:

- 1
- 2
- 3
- 4
- 5
- 6
- 7
- 8
- 9
- 10
- 11
- 12
- 13
- 14
- 15
- 16

### Question #1: Discrete Structures

Prove that every invertible function is one-to-one.

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### Question #2: Logic Design

Let the 4 bits  $ABCD$  represent a decimal digit in BCD. This function of variables  $A, B, C, D$  is required:

$$f(A, B, C, D) = \begin{cases} 1 & \text{if the decimal digit equals 8 or 9} \\ 0 & \text{if the decimal digit is less than 8} \end{cases}$$

The six non-BCD combinations of  $A, B, C, D$  are "don't cares" of  $f$ .

- Give the Truth Table for  $f$ .
  - Find a completely specified, minimal sum-of-products (MSOP) for  $f$ .
  - Write the canonical SOP for the MSOP you found in part (b).
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### Question #3: Calculus

Determine whether the following series is convergent or divergent (prove your claim):

$$\sum_{n=1}^{\infty} \frac{3^n}{(2n)!}$$

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### Question #4: Introduction to Programming

Given array  $A$  containing  $n$  points, write a function with prototype

```
double mindist(unsigned int n, point *A);
```

that either returns the minimum distance between distinct points in  $A$ , or else zero if  $n$  is less than 2, or all points in  $A$  have the same value.

You may use the functions `norm` and `diff`, whose prototypes and descriptions are:

```
double norm(point *p);           // Euclidean distance from origin to p
point *diff(point *p, point *q) // pointer to a point having value p-q
```

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**Question #5: Probability and Random Variables**

Let the continuous random variables  $X, Y$  have joint distribution

$$f_{X,Y}(x,y) = \begin{cases} 1/x & \text{if } 0 < y < x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

Compute  $E(X)$  and  $E(Y)$ .

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**Question #6: Computer Architecture and Organization**

A processor can read or write its data and instruction caches in 2 CPU cycles and from main memory in 10 CPU cycles. Assume that only one memory operation (read or write) can proceed at any time, that two cache operations can proceed at once as long as only one of them is a write, and that memory and cache operations can proceed in parallel. Further assume that instruction decoding and execution each take one cycle.

The processor has a five-stage pipeline. The five stages are Instruction Fetch, Instruction Decode, Operand Fetch, Execute, and Writeback. The Decode and Execute stages always take 1 cycle each.

What is the minimum time (in cycles) required to execute a sequence of 3 arithmetic instructions (no branching) each of which reads one operand from memory and writes one result back to memory. What is the maximum time? You may illustrate your answers with a Gantt chart.

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**Question #7: Data Structures**

- Describe – no code necessary – how a heap can be used for sorting  $n$  elements in  $O(n \log n)$  time.
  - Describe – no code necessary – how a self-balancing binary tree can be used for sorting  $n$  elements in  $O(n \log n)$  time.
  - Describe – no code necessary – how linked lists can be used for sorting  $n$  elements in  $O(n \log n)$  time.
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**Question #8: Algorithms**

Consider the following problem:

Given an integer sequence  $x_1, \dots, x_n$ , is there a nonempty sub sequence which sums to zero?

Describe – no code necessary – a dynamic programming solution based on the predicate

“A nonempty sub sequence of  $x_1, \dots, x_i$  has sum  $s$ ”

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**Question #9: Systems Programming and Operating Systems**

Suppose you are designing a program that needs to write a *log file* which will record the time at which the program was invoked as well as the command line options with which it was invoked. The log file will reside in your home directory with a name that you choose when you compile the program and which will not change. It may be invoked by any user on the system. Explain how your program can be made to write such a file while maintaining appropriate security precautions.

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**Question #10: Linear Algebra**

Consider the following collection of vectors.

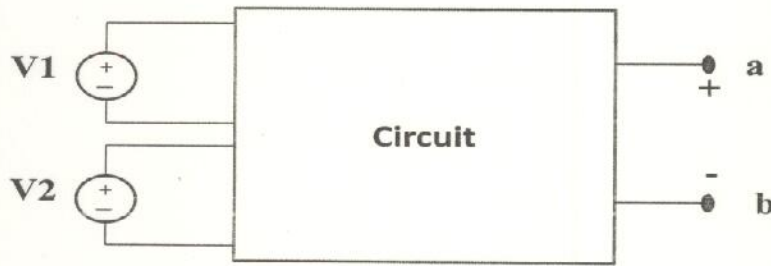
$$v_1 = \begin{bmatrix} 1 \\ -1 \\ -3 \end{bmatrix}, v_2 = \begin{bmatrix} -5 \\ 7 \\ 8 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ 1 \\ h \end{bmatrix}$$

Find the values of  $h$  for which the vectors are linearly dependent. Be sure to justify your answer.

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**Question #11: Circuits**

Consider a linear circuit consisting of one million resistors, two voltage sources with voltages  $V_1$  and  $V_2$  (see below). When  $V_1=1V$  and  $V_2=1V$ , we have  $V_{ab} = 5V$ . When  $V_1=2V$  and  $V_2=1V$ , we have  $V_{ab} = 7V$ . Assuming we set  $V_1=2V$  and  $V_2=3V$ , what is the value of  $V_{ab}$ ?



**Question #12: Signals and Systems**

The bilateral Laplace transform of a continuous-time function  $x(t)$  is defined by  $X(s) = \int_{-\infty}^{\infty} x(t)e^{-st} dt$ .

For each function below find the region in the complex  $s$  plane in which the bilateral Laplace transform integral converges.

(a)  $x(t) = \begin{cases} 1, & |t| < 1 \\ 0, & \text{otherwise} \end{cases}$

(b)  $x(t) = u(t-3)$  where  $u(t)$  is a bounded function defined by  $u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$

(c)  $x(t) = e^{5t} \cos(10t)u(t)$

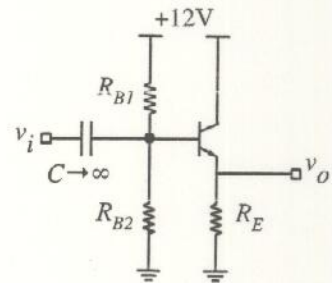
(d)  $x(t) = e^{5t} \sin(10t)u(-t)$

(e)  $x(t) = e^{5t} \sin(10t)u(-t+4)$

(f)  $x(t) = e^{-3t} \sin(10t)u(t) + e^{6t} \cos(15t)u(-t)$

**Question #13: Electronics**

For the common-collector amplifier shown here, find values for  $R_{B1}$ ,  $R_{B2}$ , and  $R_E$ , such that the output resistance is  $50 \Omega$ , the current in  $R_{B1}$  is 10 times the base current, and the DC output voltage is 6 V. Assume that the saturation current  $I_S = 10^{-15}$  A,  $\beta = 100$ , and  $V_T = kT/q = 25$  mV. Neglect the Early effect, and assume that the effect of  $R_E$  on output resistance is negligible.



### Question #14: Power Electronics

A single phase transformer with voltage rating of 2200V / 220V has the following test data:  
Short Circuit test (LV side shorted):  $V=150V$ ,  $I=4.55A$ , and  $P=215W$ .

Open Circuit test (HV side open):  $V=110V$ ,  $I=2.0A$ , and  $P=100W$ .

Find:

- 1) The winding resistance ( $R_{eq}$ ) and leakage reactance ( $X_{eq}$ ) parameters referred to the low voltage side? (50%)
  - 2) The core impedance parameters  $R_c$  and  $X_m$  referred to the high voltage side? (50%)
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### Question #15: Electromagnetics

- 1) A uniform plane wave in air with  $E = 8 \cos(\omega t - 4x - 3z) a_y \text{ V/m}$  is incident on a dielectric slab ( $z \geq 0$ ) with  $\mu_r = 1.0$ ,  $\epsilon_r = 2.5$ ,  $\sigma = 0$ . Find:
    - a) The polarization of the wave
    - b) The angle of incidence ( $\tan \theta_i = k_{ix}/k_{iz}$ ),  $k_{ix}$  and  $k_{iz}$  are the propagation constants in x and z directions.
    - c) The reflected E field
    - d) The transmitted H field.
  - 2) A  $70 \Omega$  lossless line has  $s=1.6$  and angle of reflection coefficient at the load is  $300^\circ$  degree. If the line is  $0.6\lambda$  long. Obtain
    - a)  $\Gamma$ ,  $Z_L$ ,  $Z_{in}$
    - b) The distance of the first minimum voltage from the load
    - a) Use single stub match matching.
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### Question #16: Communication Systems

Consider a conventional amplitude modulated (AM) system where we can use either a synchronous or envelope detector. Let  $v(t)$  be the input to the detector, where we can represent it in either quadrature-carrier form, or envelope-phase form. That is

$$v(t) = \begin{cases} v_i \cos 2\pi f_c t - v_q \sin 2\pi f_c t & \text{quadrature carrier form} \\ A_v(t) \cos[2\pi f_c t + \phi(t)] & \text{envelope phase form} \end{cases}$$

where  $f_c =$  carrier frequency,  $A_v(t) = \sqrt{v_i^2(t) + v_q^2(t)}$ ,  $\phi = \tan^{-1} \frac{v_q(t)}{v_i(t)}$ , etc.

We also know that the outputs of the detectors are:

$$y(t) = \begin{cases} v_i(t) - \bar{A}_v & \text{synchronous detector} \\ A_v(t) - \bar{A}_v & \text{envelope detector} \end{cases}$$

Now let's consider the performance of both detectors in the presence of multipath interference such that the AM signal going into the detector is  $v(t) = x_c(t) + \alpha x_c(t - t_d)$ , where  $x_c$  = the transmitted signal,  $\alpha x_c(t - t_d)$  is the multipath component and  $\alpha^2 < 1$ . Let's assume 100% modulation,  $A_c$  = the AM signal amplitude,  $x(t)$  represents the message,  $\mu$  = the modulation index.

Let's also note the following trig relationships:  $\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$ ,  
 $\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$ ,

$$\sin \alpha \sin \beta = \frac{1}{2} \cos(\alpha - \beta) - \frac{1}{2} \cos(\alpha + \beta), \quad \cos \alpha \cos \beta = \frac{1}{2} \cos(\alpha - \beta) + \frac{1}{2} \cos(\alpha + \beta),$$

$$\text{and } \sin \alpha \cos \beta = \frac{1}{2} \sin(\alpha - \beta) + \frac{1}{2} \sin(\alpha + \beta).$$

- Express the analytical expression for the detector input as a function of  $A_c$ ,  $x(t)$ ,  $\alpha$ ,  $t_d$ , etc.
- Derive the analytical expression for the envelope detector output; simplify as much as possible.
- Derive the analytical expression for the synchronous detector output; simplify as much as possible.
- Compare the performance of the two detectors, particularly when the delay is such that

$$\omega_c t_d = \pi/2.$$


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