DEPARTMENT OF ELECTRICAL ENGINEERING AND COMPUTER SCIENCE
UNIVERSITY OF TENNESSEE
FALL 2013 Ph.D. QUALIFYING EXAMINATION
Monday, August 19, 2013

Exam Packet Number: _____

You are allowed 4 hours to complete this exam.

- This exam is closed book and closed notes. No calculators or cell phones are allowed.
- **All your work should be done on the papers that are supplied to you. Do not write on the back of any page. Do not write any answers on this packet!**
- Be sure to put your exam packet number on each sheet that has material to be graded. Do not put your name on any sheet!
- There are 16 equally weighted problems. You are to SELECT ANY EIGHT of these to answer. You must make it very clear which eight you choose (see below). If it is not made clear by you, then the first eight problems that you attempt to answer will be graded. Circle only the eight (8) questions that you want graded below:

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1. Discrete Structures (CS311)

(a) If there are T distinct tasks and P identical processors, with $P \geq T$, in how many ways can processors be assigned to the tasks so that each task is allocated at least one processor? (We don't care which processors are assigned, only the number assigned to each different task.) You do not have to simplify your formula.

(b) In how many ways can a positive integer N be written as a sum of (at least one and at most N) positive integer summands if the order of the summands is relevant? For example, 2+3, 3+2, 5, 1+1+1+1+1 are some of the different ways of writing 5 as a sum of positive integers. You do not have to simplify your formula.

(c) The Fibonacci numbers are defined $F_0 = 0$, $F_1 = 1$ and for $n \geq 2$, $F_n = F_{n-1} + F_{n-2}$. Prove by induction that

$$\sum_{k=1}^{n} \frac{F_{k-1}}{2^k} = 1 - \frac{F_{n+2}}{2^n}.$$

2. Logic Design (ECE255)

Consider the function F(A,B,C,D) = (AB + BC)(CD + AD). For both parts, assume that you have access to both the signals and their complements.

(a) Implement F using only 2-input NOR gates.
(b) Implement the same function using only an 8:1 multiplexer.

3. Calculus (Math)

Find the absolute maximum and minimum values, if any, of the function $f(x) = x^3 - 3x + 2$ on the interval $[0,\ 2]$.

4. Programming (ECE206/CS102)

Write a function named checkout that calculates the total cost of the items in a shopping cart. It should take as arguments an array of items and the number of items in the array. It should return the total cost of the items as a floating point number. Each item is declared as a class as follows:

```
class item {
public:
   string itemName;
   int quantity;
   double costPerItem;
};
```

Your function should:

1) for each item print a line with the name of the item and the item's total cost. The item's name should be printed in a field 20 characters wide and the item's cost should be printed to 2 decimal digits of accuracy.
2) print the total cost of all the items with 2 decimal digits of accuracy
3) print "free shipping" if the total cost of the items is greater than or equal to $100, and "shipping will cost $5" otherwise.
4) return the total cost of the items

Here is sample output from the function:

```
paper: 48.90
pen: 30.00
flashdrive: 8.83
total cost = 87.73
shipping will cost $5
```

You may use Java, C++, or C. Please specify which language you use. If you use C, you may assume that the above class is a struct. If you use Java, still pass the number of items in the array, even though you could determine this number by querying the array.

5. Probability and Random Variables (ECE313)

Let X be a Gaussian random variable with mean 1 and variance 4. Assume $\Phi(x)$ denotes the cumulative distribution function (c.d.f.) of a Gaussian random variable with zero mean and variance of 1. Let $Y = e^X$. Find the c.d.f. $F_Y(y)$ of Y. Your solution should be an explicit function of y.

6. Computer Architecture (ECE355/ECE451/CS160)

Caches are widely used in computer systems to exploit locality of programs. This problem asks you how it works. Suppose that there are a series of address references given as word addresses: 0, 3, 11, 16, 21, 11, 16, 48, 16.

a) Show the hits and misses and final cache contents for a direct-mapped cache with one-word blocks and a total size of 16 words.

b) Show the hits and misses and final cache contents for a two-way set-associative cache and a total size of 16 words. Assume LRU replacement. (Hint: there is a total of eight sets in this case)

7. Data Structures (CS140)

Pick your favorite balanced search tree, and answer the following questions about it:

1. What is its name?
2. What conditions does it impose on its trees that keep them balanced?
3. What is the running time of insertion, deletion, finding an element in a tree with n elements?
4. Show an example of inserting an element into a tree that requires work to keep it balanced, highlighting how the balance is maintained.

8. Algorithms (CS302)

(a) Briefly describe how mergesort works. Assume the data to be sorted in stored in an array.

(b) State the recurrence relation that describes the worst case computational cost associated with mergesort, then solve it using "telescoping." Show details of your work --- avoid magic steps.

(c) Suppose the data to be sorted is stored in a linked list. Which part of mergesort readily supports such a set-up? Which part of mergesort becomes more cumbersome? Derive the resulting worst case computational cost.

9. Operating Systems (CS360)

(a) Explain what an *inode* is, what information it contains, and how it relates to files, links, directories and the **stat** system call.
(b) What is buffered I/O and what are its benefits? Give at least one example of buffered input, buffered output, un-buffered input and un-buffered output call that a programmer can make in a C program on Unix. Explain for each example, whether it is a system call, a C standard library call or both.
(c) As an application programmer, between buffered I/O and un-buffered I/O, which should be preferred? Justify your answer.
(d) Finally, what are the differences between system call and library call?

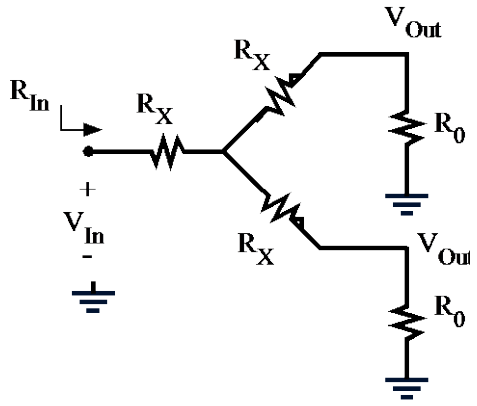
10. Linear Algebra (Math)

Consider a matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. We know that one eigenvalue of *A* is λ_1. Then, find the other eigenvalue λ_2 of A, as a function of *a, b, c, d* and λ_1. Fix λ_1, can *a, b, c, d* be any arbitrary four numbers?

11. Circuits (ECE201/202)

(a) For the circuit shown at right, find a value for R_X such that the equivalent resistance looking in as indicated is equal to R_0.

(b) If a voltage V_{In} is applied at the input, find the voltage across the two output resistors labeled R_0.



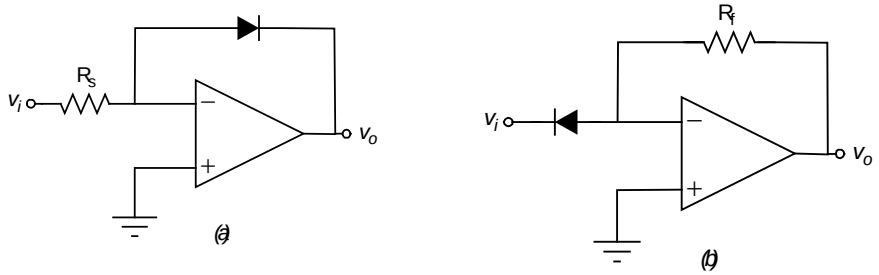
12. Signals & Systems (ECE315/316)

A signal $\mathbf{x}_i(t) = 10\,\mathrm{sinc}(20t)$ is sampled by an ideal impulse sampler at 12 Hz to form $\mathbf{x}_{is}(t) = 10\,\mathrm{sinc}(20t)\sum_{k=-\infty}^{\infty}\delta(t-k/12)$ where $\delta(t)$ is the unit continuous-time impulse. Then $\mathbf{x}_{is}(t)$ is passed through an ideal, unity-gain, lowpass filter with a cutoff frequency of 10 Hz to form $\mathbf{x}_{out}(t)$. Find the numerical signal energy of $\mathbf{x}_{out}(t)$.

$$\left\{\begin{array}{c} \mathrm{sinc}(t) = \dfrac{\sin(\pi t)}{\pi t} \overset{\mathcal{F}}{\longleftrightarrow} \mathrm{rect}(f) = \begin{cases} 1\ , |f| < 1/2 \\ 0\ , |f| > 1/2 \end{cases} \\ \sum_{k=-\infty}^{\infty}\delta(t-kT_0) \overset{\mathcal{F}}{\longleftrightarrow} \dfrac{1}{T_0}\sum_{k=-\infty}^{\infty}\delta(t-k/T_0) \\ \text{Signal Energy of } \mathrm{x}(t),\ E_x = \int_{-\infty}^{\infty}|\mathrm{x}(t)|^2\,dt = \int_{-\infty}^{\infty}|\mathrm{X}(f)|^2\,df \\ \mathrm{X}(f) = \mathcal{F}\left(\mathrm{x}(t)\right) = \int_{-\infty}^{\infty}\mathrm{x}(t)e^{-j2\pi ft}\,dt \end{array}\right\}$$

13. Electronics (ECE335/336)

For the following circuits, assume that the thermal voltage V_T = kT/q is negligible in comparison to the voltage drop across the diode and that the opamps have finite positive and negative power supplies, VDD and VSS, but are otherwise ideal. Use the exponential characteristic for the forward-biased diodes and assume that reverse-bias diode current is negligible. For both circuits, find the expression for the output voltage V_o in terms of the input voltage V_i and device parameters (R_F, R_S, I_S, VSS, VDD).



14. Power (ECE325)

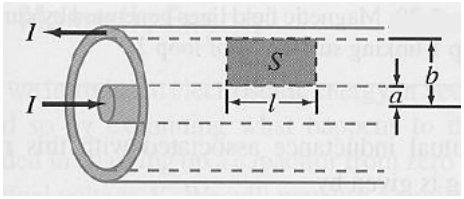
A single phase 2200V /220V transformer has the following test data:
Short circuit test (low voltage side shorted): V=150 volts, I=4.55 amps, P=215 watts Open circuit test (high voltage side open): V=120 volts, I=2.5 amps, P=200 watts

1) What are the winding resistance (R_{eq}) and leakage reactance (X_{eq}) parameters referred to the low voltage side?
2) What are the core impedance parameters R_c and X_m referred to the high voltage side?

15. Electromagnetics (ECE341)

The dimensions of a coaxial transmission line is as shown in the Figure (notice *a* and *b*), with the space filled by a dielectric with a relative permittivity ε_r and a relative permeability of 1 (meaning $\mu = \mu_0$). To find the unit length capacitance *C*' and the unit length inductance *L*', consider the line to be infinitely long.

(1) Find the electric field at a distance *r* from the axis. You may assume the inner conductor has a linear charge density (i.e. charge per length) ρ_l, while the outer conductor $-\rho_l$. You may take ρ_l as positive. **Note:** You need to find expressions for all *r*: $r < a$, $a < r < b$, and $r > b$. Use Gauss's law.



(2) Find the voltage *V* between the inner and outer conductors, and then use the relation $Q = CV$, where *Q* is charge and *C* is capacitance, to show that the unit length capacitance $C' = 2\pi\varepsilon_r\varepsilon_0 / \ln(b/a)$.

(3) Using the relation $\Phi = LI$, where Φ is the flux, *I* is the current, and *L* is the inductance, one can show that $L' = (1/2\pi)\,\mu_0 \ln(b/a)$, which you are **not** required to show. Find the phase velocity v_p, assuming $\varepsilon_r = 2.25$. (**Tips**: $1/\sqrt{\mu_0\varepsilon_0} = c = 3 \times 10^8$ m/s)

16. Communications (ECE342)

Suppose that a communication system has bandwidth *W* and transmit power *P*. We assume that the channel is an additive white Gaussian noise (AWGN) one. The noise power density is N_0. Then, what is the maximum reliable transmission rate of this communication system? If the system is given an infinite bandwidth, will the maximum transmission rate also be infinite? If not, what is the maximum rate?