Exam Packet Number: ________________________________

You are allowed 4 hours to complete this exam.

- This exam is closed book and closed notes. No calculators or cell phones are allowed.
- All your work should be done on the papers that are supplied to you. Do not write on the back of any page. Do not write any answers on this packet!
- Be sure to put your exam packet number on each sheet that has material to be graded. Do not put your name on any sheet!
- There are 16 equally weighted problems. You are to SELECT ANY EIGHT of these to answer. You must make it very clear which eight you choose (see below). If it is not made clear by you, then the first eight problems that you attempt to answer will be graded. Circle only the eight (8) questions that you want graded below:

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
1. Discrete Structures (CS311)

Let $S = \{0, 1, \ldots, 9\}$.

a. How many functions $f: S \times S \to S$ are there?

b. How many of these functions satisfy $f(1, 2) = 0$?

c. How many functions in part (b) have 0 as an identity, that is, $f(0, x) = x = f(x, 0)$?

d. How many functions in part (b) have an identity (not necessarily 0)?

e. How many commutative functions $f: S \times S \to S$ are there, that is, functions for which $f(x, y) = f(y, x)$?

f. How many of the commutative functions satisfy $f(1, 2) = 0$?

g. How many commutative functions have an identity?

Suppose $S = \{0, 1, \ldots, N\}$, with $N > 2$. Answer each of preceding questions (a)–(g) for this set.

2. Logic Design (ECE255)

(a) Analysis (25%)

Derive the two D-FF input equations. First write them exactly as the circuit is drawn, then apply Boolean rewrites to obtain sum-of-product forms based entirely on AND, OR, and NOT gates.

(b) Analysis (25%)

Derive first a state table for the circuit, then extract the corresponding state diagram. Label everything appropriately.

(c) Design (50%)

Replace the lower D-FF labeled $Q_0$ including its associated input circuitry with a JK-FF.

(c1) Expand the state table from (b) with the needed JK columns.

(c2) Perform K-map based design for the JK-FF inputs. That is, obtain the simplest sum-of-products expressions possible.

(c3) Sketch the resulting circuit.
3. Calculus (Math)

You need to solve all three problems.

1) The velocity of an object is given by
\[ v(t) = \frac{1}{\pi} \left( \frac{1}{2} + \sin(\pi t) \right) \text{ meters per second,} \]
where \( v(t) > 0 \) for \( t < \frac{\pi}{6} \) and \( v(t) < 0 \) for \( t > \frac{\pi}{6} \). Find the total distance traveled in meters in the first 1.5 seconds.

2) Determine if the following vectors are parallel, orthogonal (i.e. perpendicular), or neither.
   a) \( \vec{a} = \langle 6, -2, -1 \rangle \) and \( \vec{b} = \langle 2, 5, 2 \rangle \)
   b) \( \vec{u} = 2\mathbf{i} - \mathbf{j} \) and \( \vec{v} = -\frac{1}{2}\mathbf{i} + \frac{1}{4}\mathbf{j} \)

3) The LCR-circuit is an electrical circuit consisting of a resistor of resistance \( R \), a coil of inductance \( L \), a capacitor of capacitance \( C \). Assuming there is no voltage source in the circuit, a differential equation model of an LCR-circuit where \( Q \) is the charge across the capacitor, is given by
\[ L \frac{d^2Q}{dt^2} + R \frac{dQ}{dt} + \frac{1}{C} Q = 0 \]
Let \( L = 0.2 \) Henry, \( C = 10^{-5} \) Farads, and \( R = 3 \times 10^2 \Omega \). Let there be an initial charge across the series capacitor of \( 10^{-6} \) Coulomb and let there be no initial current. Find the charge across the capacitor as a function of time.
Write a function named movingAvg that computes the moving average of a sequence of numbers. The problem specification is as follows:

1. movingAvg is a void function.
2. movingAvg takes four parameters:
   a. an array of doubles that contains the sequence of numbers
   b. an array of doubles that contains the computed moving average. This array will hold the result of your function.
   c. an integer representing the size of the two arrays
   d. an integer representing the length of the “window” used for the moving average.
3. You may use either C, C++, or Java for your solution. Please indicate which language you are using. Note that if you use Java, the parameter representing the size of the two arrays is not strictly necessary, but you should still include it.
4. Do not show any other code, such as main. Only show the function.

As an example, if your array of numbers is:

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>1.0</td>
<td>2.0</td>
<td>3.0</td>
<td>4.0</td>
<td>5.0</td>
<td>6.0</td>
<td>7.0</td>
</tr>
</tbody>
</table>

and your “window” is 4, then the array that contains your moving average should have the following values when your function exits:

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>3.5</td>
<td>4.5</td>
<td>5.5</td>
<td>6.5</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

The first moving average is 2.5 because the average of the first four entries in your array of numbers is 2.5. The second moving average is 3.5 because you “slide” the window right by 1 element and take the average of the numbers in entries 1-4 of your array of numbers. The entries for 5-7 in the moving average array are unknown because you cannot compute a moving average of length 4 for them (i.e., your function would stop as soon as it finished computing the average of entries 4-7 in your array of numbers).
5. Probability and Random Variables (ECE313)

You have two biased coins. Coin A comes up heads with probability 0.25 and coin B comes up heads with probability 0.75. You are not sure which coin is which, so you choose one coin at random and flip it. If the flip is heads, you guess that the flipped coin is B, otherwise you guess that the flipped coin is A. What is the probability that your guess is correct?


Media applications that play audio or video files are part of a class of workloads called "streaming" workloads, i.e., they bring in large amounts of data but do not reuse much of it. Consider a video streaming workload that access a 512KB working set sequentially with the following address stream:

0, 4, 8, 12, 16, 20, 24, 28, 32......

(1) Assume a 64KB direct-mapped cache with a 32-byte line. What is the miss rate for the address stream above? How is this miss rate sensitive to the size of the cache or the working set? How would you categorize the misses this workload is experiencing, based on the model of three types of cache misses?

(2) Re-compute the miss rate when the cache line size is 16 byte, 64 bytes, and 128 bytes? What kind of locality is this workload exploiting?

(3) Prefetching is a technique that leverages predictable address patterns to speculatively bring in additional cache lines when a particular cache line is accessed. One example of prefetching is a stream buffer that prefetches sequentially adjacent cache lines into a separate buffer when a particular cache line is brought in. If the data is found in the prefetch buffer, it is considered as a hit and moved into the cache and the next cache line is prefetched. Assume a two-entry stream buffer and assume that the cache latency is such that a cache line can be loaded before the computation on the previous cache line is completed. What is the miss rate for the address stream above?
This question concerns AVL trees. For this question, you are going to start with the tree below. In this tree, the circles are nodes, and the triangles are all valid AVL trees whose keys start with the letter in the tree.

I am going to give you six scenarios. In each, I tell you the heights of the trees in the in triangles, and I tell you what action has just occurred. The state of the tree is before the rebalancing. You need to tell me what rebalancing operations need to be performed.

Choose your answers from the multiple choices that follow the scenarios.

<table>
<thead>
<tr>
<th>A</th>
<th>C</th>
<th>E</th>
<th>G</th>
<th>I</th>
<th>K</th>
<th>M</th>
<th>O</th>
<th>Scenario</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>4</td>
<td>6</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>7</td>
<td>6</td>
<td>1. We have inserted a new node into M</td>
</tr>
<tr>
<td>47</td>
<td>46</td>
<td>45</td>
<td>46</td>
<td>44</td>
<td>45</td>
<td>45</td>
<td>45</td>
<td>2. We have deleted a node from O</td>
</tr>
<tr>
<td>83</td>
<td>82</td>
<td>83</td>
<td>83</td>
<td>84</td>
<td>85</td>
<td>83</td>
<td>84</td>
<td>3. We have deleted a node from E</td>
</tr>
<tr>
<td>17</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>16</td>
<td>17</td>
<td>16</td>
<td>17</td>
<td>4. We have inserted a new node into G</td>
</tr>
<tr>
<td>35</td>
<td>34</td>
<td>33</td>
<td>33</td>
<td>34</td>
<td>33</td>
<td>33</td>
<td>34</td>
<td>5. We have deleted a node from G</td>
</tr>
<tr>
<td>60</td>
<td>59</td>
<td>61</td>
<td>60</td>
<td>59</td>
<td>58</td>
<td>59</td>
<td>58</td>
<td>6. We have inserted a new node into E</td>
</tr>
</tbody>
</table>
Multiple Choice Answers:

- a: Do nothing
- b: Single rotate about B
- c: Double rotate about B
- d: Single rotate about D
- e: Double rotate about D
- f: Single rotate about F
- g: Double rotate about F
- h: Single rotate about H
- i: Double rotate about H
- j: Single rotate about J
- k: Double rotate about J
- l: Single rotate about L
- m: Double rotate about L
- n: Single rotate about N
- o: Double rotate about N

Just so we're not confused, I'm going to answer scenario 1 for you. We have just inserted a node into tree M, and its height after the insertion is 7. The resulting tree is drawn below, and I've put all of the heights in all of the nodes. You can see that this tree is a valid AVL tree, so we don't need to do any rebalancing. The answer is “a: Do nothing.”
8. Algorithms (CS302)

An "ABC-10" string is a string that is ten characters long, and is made up of the characters A, B, and C. Write an efficient program that prints out all ABC-10 strings that contain exactly one character C.

9. Operating Systems (CS360)

Both problems need to be answered.

1. (a) Explain the functionalities of fopen/fclose, fread/fwrite, scanf/fscanf, and printf/fprintf. (b) Explain the functionalities of open/close, read/write. (c) For the calls listed in (a) and (b), explain which ones are library calls and which are system calls, and which ones implement buffered I/O and which ones implement un-buffered I/O? (d) As an application programmer, between library calls and system calls, which should be preferred? Between buffered I/O and un-buffered I/O, which should be preferred? Justify your answer.

2. Explain how pipe and socket facilitate inter-process communication in Unix. In addition, please describe how pipe is used for inter-process communication when the shell gets the following command:

```
cat grades.dat | grep A | sort | more
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10. Linear Algebra (Math)

The range of a matrix A means the set of all possible linear combinations of its column vectors. Prove that \( A^T A \) and \( A^T \) have the same range, where A is an \( n \)-by-\( n \) real matrix and \( T \) means transpose.
11. Circuits (ECE300)

For the infinite resistor ladder shown, find $R_\infty$, the equivalent resistance looking in from the driving terminal as shown. Your answer should be expressed in terms of the unit resistor $R$.

![Resistor Ladder Diagram]

12. Signals & Systems (ECE315/316)

Consider the linear time invariant (LTI) system described by the following differential equation:

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = x(t)$$

Given that $e^{-at}u(t) \mapsto \frac{1}{a+j\omega}$ where $Re\{a\} > 0$, use Fourier transforms to determine:

(a) The frequency response $H(j\omega)$ of the system.

(b) The impulse response $h(t)$ of the system.

(c) The response $y(t)$ to the input $x(t) = e^{-4t}u(t)$. 
13. Electronics (ECE335/336)

For the CMOS inverter shown at right, assume the NMOS and PMOS have a length of 0.7µm and \( V_{tn} = -V_{tp} = V_{th} \).

a) Draw the voltage transfer characteristic and indicate the mode of operation for each transistor throughout the curve.

b) If \( \mu_n \approx 3 \times \mu_f \) and the NMOS has a gate width of 2µm, what should the gate width of the PMOS be in order for the transition to occur at \( V_{tn} = V_{DD}/2 \)? Why?

c) How will the transition voltage shift if the PMOS and NMOS have the same width?

14. Power (ECE325)

A balanced, three-phase, 60 Hz induction motor delivers rated output power. The rotor speed is 1160 rpm. The rotor circuit frequency (i.e., the slip frequency or \( f_2 \)) is 2Hz. Determine:

1) The synchronous speed in rpm. (20%)
2) The number of poles. (20%)
3) The speed of the rotor relative to the stator rotating field. (20%)
4) The speed of the rotor relative to the rotor rotating field. (20%)
5) The speed of the rotor rotating field relative to the stator structure. (20%)
15. Electromagnetics (ECE341)

As shown in Fig. (a), two infinitely large ideal conductor slabs are a distance $2a$ apart. Both slabs are $b - a$ thick. The slab on the right side ($x > 0$) has a charge per area, $\sigma$, while the one on the left side has an equal but opposite charge. Take $\sigma$ as positive. In Fig. (a), complete the plots of the field $E(x)$ and the potential $V(x)$ to visualize them as functions of $x$ for all $x$ (including $|x| > b$). For convenience, you may set the potential at $x = 0$ to be zero, i.e. $V(0) = 0$. Specify the types of functions (constant, linear, quadratic, exponential, etc) for $E(x)$ and $V(x)$ in each segment, although exact analytical expressions are not required.

As shown in Fig. (b), an infinitely large slab of dielectric with a dielectric constant $\varepsilon_r$ is at distance $2a$ from another infinitely large slab of dielectric, with the same dielectric constant $\varepsilon_r$. Both slabs are $b - a$ thick. The dielectric slab on the right side ($x > 0$) has a uniform volume charge density $\rho$, while the one on the left side has a charge density $-\rho$. Take $\rho$ as positive. In Fig. (b), complete the plots of the field $E(x)$ and the potential $V(x)$ to visualize them as functions of $x$ for all $x$ (including $|x| > b$). For convenience, you may set the potential at $x = 0$ to be zero, i.e. $V(0) = 0$. Specify the types of functions (constant, linear, quadratic, exponential, etc) for $E(x)$ and $V(x)$ in each segment, although exact analytical expressions are not required.

Take $\sigma = (b - a)\rho$. In which of the two cases [Fig. (a) and Fig. (b)] is the voltage drop $V(b) - V(-b)$ larger?

(a) A pair of charged conductor slabs  
(b) A pair of uniformly charged dielectric slabs
16. Communications (ECE342)

(1) Explain what DSB, SSB and VSB are. Sketch example diagrams of the resulting spectrum from each when the 2 kHz audio signal below is modulated onto a 5 MHz high frequency carrier.

(2) Signal \( x(t) = \cos 2\pi 100t + 4\cos 2\pi 200t \) is transmitted using DSB with carrier frequency 5000 Hz. Sketch the positive-frequency line spectrum. Repeat it when SSB is used.